Chapter 8
Bearing Capacity of Floating Ice Sheets

8-1. Introduction

In cold regions, ice covers on rivers, lakes, and seas are often used as temporary roads, bridges, airfields, and construction platforms. For these uses, it is important that there be a sufficient margin of safety between the breakthrough loads and the actual loads placed on a floating ice sheet. This chapter discusses the bearing capacity of floating ice sheets of any given thickness, or how to determine the required ice thickness for a given load.

a. The thickness of an ice sheet can be either measured by drilling holes in it or estimated for a location from atmospheric temperature data and the theoretical formulas presented in Chapter 2 of this manual.

b. At times, the actual ice thickness at a location is less than the required minimum ice thickness for a given load. A common practice to increase ice thickness is to plow the snow off the path chosen for a road. Removing the snow allows the ice sheet to grow faster, and thus increases the bearing capacity at those locations. However, the plowed snow banks become loads on the ice sheet, and they provide extra insulation to the ice at those locations, which retards ice thickness growth. Both effects can decrease safety.

c. Another common practice to increase ice thickness is by flooding and freezing in a prescribed series of steps. The literature gives examples of thickening a natural ice cover by flooding and freezing in thin layers, after which the ice has held loads as heavy as 5 meganewtons (MN) (500 tons) for 30 days and longer. However, such operations need careful planning and execution.

8-2. Bearing Capacity of Ice Blocks

The main source of bearing capacity for a floating ice sheet is its buoyancy, or the hydrostatic pressure on its bottom, because the density of ice is less than the density of liquid water. For a centrally placed or a uniformly distributed load P on an area A of an ice block (Figure 8-1), the equilibrium equation of the forces in the vertical direction is given by

$$P + Ah\gamma_i = Az\gamma_w \qquad \text{for } z < h \tag{8-1}$$

where

γ_i = specific weight of ice
γ_w = specific weight of water
h = ice thickness
z = depth of submergence.

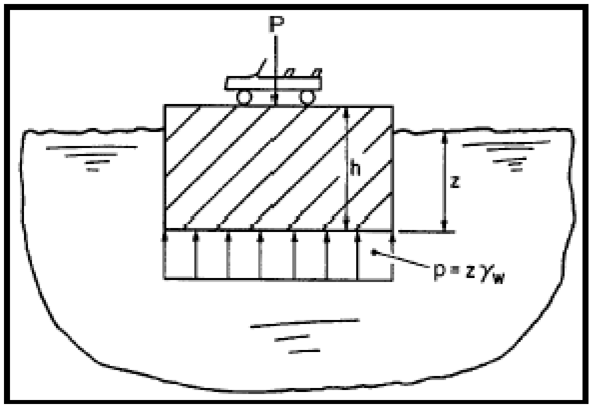

Figure 8-1. Equilibrium of forces on an ice block.

When $P = 0$, we get the depth of submergence z_o under no load, and it is given by:

$$z_o = (\gamma_i/\gamma_w)h.$$

The difference $h - z_o$ is known as the freeboard, which is associated with the bearing capacity of an ice block. When $z = h$, we get the maximum load P_{max} that can be placed on the ice block without submergence, and it is given by:

$$P_{max} = Ah(\gamma_w - \gamma_i). \tag{8-2}$$

a. When the resultant of the load is not centrally placed on the ice block, the ice block will tilt. This will result in a linearly varying pressure p at the bottom surface of the block. The bearing capacity for this case may be determined as done previously, except that now, in addition to vertical equilibrium, the moment equilibrium has to be considered. Note that when the eccentricity of the load resultant exceeds a certain limit, namely when the loading moment is larger than the restoring moment, the ice block will tip over. When the load is dynamic, the analysis is more involved. Then, the equations of motion for the ice block have to be coupled with the dynamic equations for the fluid base.

b. To illustrate the use of the above equations, let us determine the bearing capacity of an ice block having a thickness of 1 meter (3.28 feet) and an area of 10 meters^2 (107.6 feet^2). The specific weights of water and ice are given by

$$\gamma_w = \rho_w g$$

$$\gamma_i = \rho_i g$$

where

ρ_w = density of water
ρ_i = density of ice
g = gravitational acceleration.

Assuming $\rho_w = 1000$ kg/m^3, $\rho_i = 918$ kg/m^3 and $g = 9.81$ m/s^2 (or, $\rho_w g = 62.4$ lbf/ft^3 and $\rho_i g = 57.3$ lbf/ft^3), we get the bearing capacity of the ice block $P_{max} = 8.04$ kN (1808 lbf) from Equation 8-2.

c. Another example to illustrate the use of Equation 8-2 is to find the area of a 0.6-meter-thick (2-feet-thick) ice block needed to safely carry a load of 13.34 kN (3000 lbf). Using the same values for specific weights, we get

$$A = 13340/\{0.6 \times (1000 - 918) \times 9.81\} = 27.65 \text{ m}^2 \ (297 \text{ ft}^2)$$

which is a square area of about 5.26 meters (17.25 feet) per side.

8-3. Bearing Capacity of Floating Ice Sheets

When a large floating ice sheet is loaded vertically over an area, the deformation of the ice in the vicinity of the applied load causes slightly higher water pressure $p(x,y)$ under the ice sheet than the pressure p_o ($=\rho_i g h$) at farther distances (Figure 8-2). With Archimedes' principle, it can be shown that, for large ice sheets, the applied load is equal to the weight of displaced water caused by the deformation of the floating ice sheet. For quasi-static loading, this condition of equilibrium is satisfied at all times. Thus, the buoyancy force supports the load, and the ice sheet merely deforms to distribute the load over a large area. The deformation of the ice sheet generates stresses that can lead to its failure, and thus destroy the ability of the ice sheet to distribute the load.

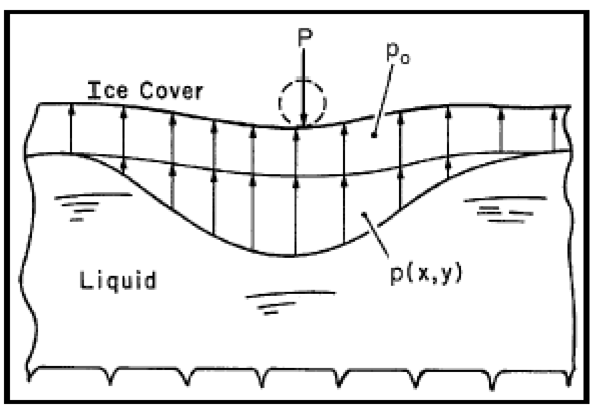

Figure 8-2. Hydrostatic pressure from a load on an floating ice sheet.

a. Because floating ice sheets exist at temperatures close to ice's melting point, it is a common experience that ice responds to applied loads by elastic and creep deformations simultaneously. During uniaxial tests at low strain rates (below 10^{-4} s^{-1} in compression and 10^{-5} s^{-1} in tension), ice deforms mostly by creep, and creep strains are generally larger than elastic strains. At high strain rates during uniaxial tests, most of the deformation in ice is elastic (because it takes time to develop creep deformation), and the failure of ice specimens is by fracture. The interplay between creep and elastic deformations, coupled with the low fracture energy required to propagate a crack, causes ice to fail in both the ductile and brittle manners with a strong dependence on strain rate, making predictions of the bearing capacity of floating ice covers very complex.

b. Because of the creep and elastic deformations and the inertial effects of underlying water, the loading on a floating ice sheet can be categorized as one of the following three types:

(1) Short-term loads, such as those imposed by slowly moving vehicles or by the placement of a load by a crane for a short time.

(2) Moving loads that are fast enough to excite the ice–water system to deflect much more than would be the case for a static load.

(3) Long-term loads, such as those imposed by parked vehicles, stored material, or drilling rigs.

In the following paragraphs, each of the three types of loading is separately discussed because of significant differences in the response of the ice sheet to these loads.

8-4. Analytical Methods for Short-Term Loads

Under this type of loading, ice behaves as an elastic, brittle material, and the ice–water inertial forces are negligible. The floating sheet can be thought of as an elastic plate resting on an elastic foundation, and its deflection is governed by the differential equation

$$D\nabla^4 w + \gamma_w w = q \tag{8-3}$$

where

D = $Eh^3/[12(1-\nu^2)]$ (flexural rigidity of the plate)
E = effective elastic modulus of ice
h = ice thickness
ν = Poisson's ratio for ice
∇^4 = biharmonic operator [e.g., $(\partial^4/\partial x^4) + 2(\partial^4/\partial x^2\partial y^2) + (\partial^4/\partial y^4)$ in Cartesian coordinates]
w = vertical deflection of a point on the ice sheet
γ_w = specific weight of water
q = load per unit area on the ice sheet.

Equation 8-3 leads to the definition of characteristic length:

$$L=(D/\gamma_w)^{1/4}. \tag{8-4}$$

Figure 8-3 shows plots of L versus h for various values of the effective elastic modulus E, for a value of Poisson's ratio $\nu = 0.3$.

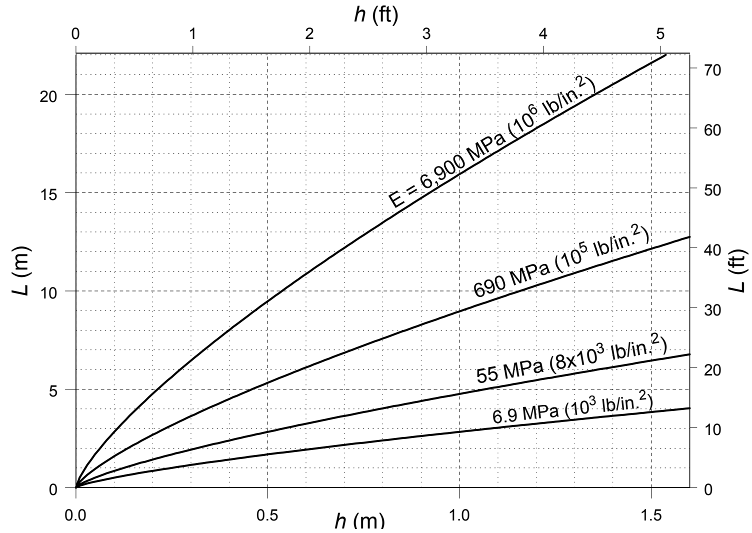

Figure 8-3. Plots of *L* versus *h* for various values of *E*.

a. A solution for Equation 8-3, which satisfies a given set of boundary conditions, can be obtained for a particular loading q, and the stresses can be obtained from the solution. The maximum tensile stress σ_{max} at the bottom of an infinite plate on elastic foundations is given by:

$$\sigma_{max} = CP/h^2 \qquad (8\text{-}5)$$

where

P = total, downward acting load uniformly distributed over a circular area of radius a
h = plate (ice) thickness
C = $0.275(1 + \nu) \log_{10} \{(Eh^3)/(\gamma_w b^4)\}$
b = $(1.6a^2 + h^2)^{1/2} - 0.675h$, when $a < 1.724h$, or $b = a$, when $a > 1.724h$.

The coefficient C is obtained from the theory of thick plates, and it does not go to infinity for concentrated loads as in the case of results from the theory of thin plates. Figure 8-4 shows plots of C versus a/L for $\nu = 0.3$ and various ratios of h/L. As shown in Figure 8-4, the maximum stress σ_{max} decreases with increasing radius a for the same load and ice thickness.

b. If a load P is applied uniformly over a square area, a by a, at the edge of a semi-infinite floating ice sheet, the maximum tensile stress can be obtained by Equation 8-5, but the constant C in Figure 8-4 is much higher than that for infinite ice sheets. If there are any wet cracks in the ice sheet, it should be treated as semi-infinite. If a load moves onto the edge of a floating ice sheet, care must be exercised to make sure that it is not large enough to create a crack at and perpendicular to the edge. The plot in Figure 8-4 also shows a decrease in the maximum stress at the edge if the load is distributed over a larger area.

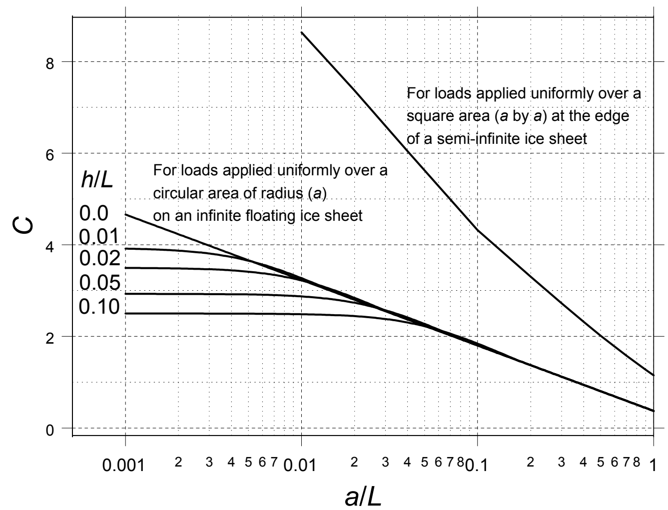

Figure 8-4. Plots of *C* versus *a*/*L* for various values of *h*/*L*.

c. To superpose the stresses from many loads on an infinite floating ice sheet, use the following procedure. The stress σ caused by a load applied at a distance x from the point of loading is given by:

$$\sigma = \sigma_{max} \exp\left(-\frac{x}{0.691L}\right) \tag{8-6}$$

The total stress at a point attributable to loads $P_0, P_1, P_2, ..., P_n$ located at distances $x_0(= 0), x_1, x_2,..., x_n$ is obtained at the point x_0 by adding the stress contributions from all loads:

$$\sigma_{total} = \sigma_0 + \sum_{i=1}^{n} \sigma_i \exp\left(-\frac{x_i}{0.691L}\right) \tag{8-7}$$

d. A safe, proven value of the maximum stress of 550 kPa (80 psi) has worked for the designs of floating platforms of sea ice, but a value of 690 kPa (100 psi) can be assumed for freshwater ice. Field experience with drilling platforms suggests the following values of the effective elastic modulus: 690 MPa for calculations of deflections and stresses immediately after placement of a load and 55 MPa for calculations of deflections at a long time after placement of a load (these values of effective elastic moduli are close to those measured by 3-meter-long [10 foot-long] strain gages used at five levels in a 7-meter-thick [22-foot-thick] ice platform). The maximum tensile stress, which depends on the distributions and distances of the loads from each other, is the immediate elastic response to a load placement, and it decreases with the passage of time because of the creep deformation of ice.

e. To illustrate the above procedure, let us determine the load-carrying capacity of a 30.5-centimeter-thick (12-inch-thick) freshwater ice sheet, assuming that the flexural strength and effective elastic modulus of ice are, respectively, 690 kPa (100 psi) and 690 MPa (10^5 psi) and that the load is distributed over a circular area of radius 100 centimeters (40 inches). From Figure 8-4, we get a value of $L = 3.7$ meters (12 feet) for $h = 0.305$ meters (12 inches) and $E = 690$ MPa (10^5 psi). For values of $a/L = 0.1/3.7 = 0.27$ and $h/L = 0.305/3.7 = 0.083$, we get a value of $C = 1.19$. Substituting values of $C = 1.19$, $\sigma_{max} = 690$ kPa (100 psi) and $h = 0.305$ meters (12 inches) in Equation 8-5, we get an estimate of the safe load $P = 54.11$ kN (12160 lbf).

f. To see the effect of load distribution on the bearing capacity, let us now determine the load-carrying capacity of the same ice sheet as in the above example, except that the load is distributed over a circular area of radius 10 centimeters (4 inches) instead of 100 centimeters (40 inches). The value of $L = 3.7$ meters (12 feet), as given above. For values of $a/L = 0.1/3.7 = 0.027$ and $h/L = 0.305/3.7 = 0.083$, we get a value of $C = 2.5$. From Equation 8-5, we get an estimate of the safe load $P = (690\text{ kN m}^{-2})(0.305\text{ m})^2/2.5 = 25.67$ kN (5772 lbf), which is less than half the value of safe load obtained above.

g. To estimate a load P that is uniformly distributed over a square area (20×20 centimeters [8×8 inches]) at the edge of a semi-infinite ice sheet, we get a value of C equal to 5.5 for the value of $a/L = (0.2/3.7) = 0.054$. The safe load P placed at the edge of a semi-infinite ice sheet is only 11.67 kN (2624 lbf) for $\sigma_{max} = 690$ kPa (100 psi).

h. The analytical methods for short-term loads presented in this paragraph compute the maximum tensile stress immediately after placement of a load on a floating ice sheet, and the use of these procedures can only lead to prediction of the loads to cause the first crack in the sheet. If the elastic stress exceeds the tensile strength, radial cracks form around the load. If the load on an ice sheet continues to increase, several circumferential cracks form before breakthrough takes place. For safe placement of loads during any operation on floating ice sheets (i.e., to prevent radial cracks from forming under the load), plans should include estimating the maximum tensile stresses, which should not exceed the tensile strength of the ice. This is a conservative approach to estimating the bearing capacity of floating ice sheets, because the breakthrough loads are generally higher than the load necessary to cause the first crack in the ice sheet.

8-5. Empirical Methods for Short-Term Loads

Data compiled on the failure loads and thicknesses of ice covers during logging and other operations indicate that breakthrough loads depend on the square of the ice thickness. By having an adequate factor of safety, a safe short-term load on a floating ice sheet can be obtained from the breakthrough loads. A load obtained by this procedure may produce radial cracks in the ice sheets, but the wedging action of a radially cracked ice sheet supports the load for a short duration of time.

a. A simple empirical formula for the loads created by single vehicles is

$$P = Ah^2 \tag{8-8}$$

where

P = allowable load
h = effective ice cover thickness

A = coefficient that depends on the quality of the ice, the ice temperature, the geometry of the load, the kind of units used, and the factor of safety.

b. To ensure safe movement of single vehicles crossing lake or river ice at temperatures below 0°C (32°F), the straightforward and practical formulas $P = h^2/16$ or $h = 4\sqrt{P}$ have been used for decades. These formulas are for English units in which P is in tons (2000 lbf) and h is in inches. Although not strictly equivalent, similar practical formulas for SI units are $P = h^2/100$ or $h = 10\sqrt{P}$, where P is in metric tons (1000 kgf, or 2205 lbf) and h is in centimeters, and $P = h^2$ or $h = \sqrt{P}$, where P is in meganewtons (MN) and h is in meters. All these formulas are for black ice below 0°C (32°F), and appropriate adjustments to thicknesses to account for snow ice should be computed as given below. The following are illustrative examples of Equation 8-8.

c. Determine the allowable load of an ice cover with the smallest ice thickness h = 25.4 centimeters (10 inches).

$$P = \frac{h^2}{16} = \frac{10^2}{16} = 6.25 \text{ tons.}$$

In metric units, this is

$$P = \frac{25.4^2}{100} = 6.45 \text{ metric tons.}$$

d. Determine the smallest ice thickness needed to safely carry one person of weight P = 200 lbf = 0.1 ton (90.7 kgf = 0.0907 metric ton).

$$h = 4\sqrt{0.1} = 1.26 \text{ inches}$$

Expressed in metric units, the required thickness is

$$h = 10\sqrt{0.0907} = 3 \text{ centimeters.}$$

e. Based on Equation 8-8, Table 8-1 lists the safe minimum values of ice thickness for tracked and wheeled vehicles on clear, sound ice. The last column of Table 8-1 lists the safe distance that should be maintained between vehicles to avoid superposition of stresses from two loads. These distances are about 100 times the required minimum ice thickness. For an ice thickness greater than the minimum required thickness, the spacing between the loads can be reduced. When driving a vehicle on an ice sheet, checking the ice thickness at regular intervals along the intended path is recommended. This should be done every 45 meters (150 feet), or more frequently, if the ice thickness is quite variable. There are several additional points to consider.

(1) If white, bubble-filled ice makes up part of the ice thickness, this part should be considered equivalent to half as much clear ice. For example, if a 76.2-centimeter-thick (30-inch-thick) ice sheet is composed of 25.4 centimeters (10 inches) of white ice and 50.8 centimeters (20 inches) of clear ice, the white ice should only be considered as 12.7 centimeters (5 inches) thick, giving an equivalent thickness of clear ice to be 50.8 + 25.4/2 = 63.5 centimeters (20 + 10/2 = 25 inches) for the computation of the safe load on that ice sheet.

(2) If there has been a large snowstorm, the snow represents a new load on the ice. If the new snow is sufficiently heavy, it will depress the whole ice sheet to a level where the top surface of the ice sheet is below the water level. Water then usually seeps through the cracks in the ice sheet and saturates the lower layers of the snow cover. Stay off the ice sheet until this slush freezes completely. When that happens, the frozen slush becomes an added thickness of white ice.

(3) Contrary to what many think, a rapid and large drop in air temperature causes an ice sheet to become brittle, and it may not be safe to use the ice sheet for 24 hours.

(4) If the air temperature stays above freezing for 24 hours or more, the ice begins to lose its strength, and the values given in Table 8-1 do not represent safe values. This becomes the general condition during springtime. No quantitative guidance can be offered for this situation. When this happens, any ice cover is unsafe for any load.

Table 8-1
Approximate Ice Load-Carrying Capacity (Note: Read the text before using this table)

Type of Vehicle	*Total Weight Metric tons (tons)*	*Necessary thickness* at average ambient temperatures for three days cm (in.)*		*Distance between vehicles m (ft)*
		0 to –7°C (32 to 20°F)	*–9°C and lower (15°F and lower)*	
Tracked	6 (6.6)	25.4 (10)	22.9 (9)	15.2 (50)
	10 (11.0)	30.5 (12)	27.9 (11)	19.8 (65)
	16 (17.6)	40.6 (16)	35.6 (14)	24.4 (80)
	20 (22.0)	45.7 (18)	40.6 (16)	24.4 (80)
	25 (27.6)	50.8 (20)	45.7 (18)	30.5 (100)
	30 (33.1)	55.9 (22)	48.3 (19)	35.1 (115)
	40 (44.1)	63.5 (25)	55.9 (22)	39.6 (130)
	50 (55.1)	68.6 (27)	63.5 (25)	39.6 (130)
	60 (66.1)	76.2 (30)	71.1 (28)	45.7 (150)
Wheeled	2 (2.2)	17.8 (7)	17.8 (7)	15.2 (50)
	4 (4.4)	22.9 (9)	20.3 (8)	15.2 (50)
	6 (6.6)	30.5 (12)	27.9 (11)	19.8 (65)
	8 (8.8)	33.0 (13)	30.5 (12)	32.0 (105)
	10 (11.0)	38.1 (15)	35.6 (14)	35.1 (115)

* Freshwater ice.
When the temperature has been 0°C (32°F) or higher for a few days, the ice is probably unsafe for any load.

8-6. Moving Loads

a. When a load moves on an ice sheet fast enough (more than 15 km/hr or 10 mph), the inertia forces generated by the movement of the ice sheet and the underlying water modify the deflections and stresses obtained from Equation 8-3 for the short-term load. For a slow-moving load, the deflection bowl in the ice sheet moves with it, and the underlying water must continually be moved aside by the bowl in a way similar to a shallow-draft boat. As for a boat, movement of the deflection bowl generates waves in the ice–water system. If the celerity of these waves is the same as the vehicle speed, the deflection and the stresses in the ice sheet are amplified, similar to resonance in an oscillating system. The critical speed at which such amplifications take place depends on the water depth H and the characteristic length L of the floating ice sheet. For deep water ($H>L$), the critical speed $u_c = 1.25\ (gL)^{1/2}$, where g is the gravitational acceleration. For shallow water ($H<L$), the critical speed $u_c = (gH)^{1/2}$. The critical speed u_c depends on the characteristic length L, which depends on the ice thickness. However, it is independent of the ice thickness in shallow water, in which water depth H is less than the characteristic length L. Because of the amplification of deflections and stresses in the ice sheet, vehicles should not approach the critical speeds.

b. As an example, let us consider 0.305-meter-thick (1-foot-thick) ice sheet, and its effective elastic modulus is 690 MPa (10^5 psi). From Figure 8-3, we get the value of characteristic length L equal to 3.7 meters (12 feet). For deep water, the critical speed of the ice–water system $u_c = 1.25(gL)^{1/2} = 7.5$ m/s (27 km/h or 16.8 mph). For water depth equal to 2 meters (shallow water), the critical speed $u_c = (gH)^{1/2} = 5.5$ m/s (20 km/h or 12.4 mph).

8-7. Long-Term Loads

When a load is placed for a long time on a floating ice sheet, there is an immediate elastic deformation of the ice, followed by permanent creep deformation. The long-term effect of creep deformation is that the vertical deflection of the loaded area increases with time, and the deflection rate is a nonlinear function of the applied load. If the load is not large, the displacement rate is small, leading to safe placement of the load for a long time. However, a high deflection rate for a large load may lead to the failure of ice sheets after a certain time. There is a need to determine the duration of time that a particular load can safely be placed on an ice sheet.

a. For time-dependent deflections less than the freeboard, elastic equations (e.g., Equation 8-3) describe the deflections of the ice sheet in the vicinity of a load, but the characteristic length, which depends on the elastic modulus E, decreases continuously with time. The maximum elastic deflection δ under a load P is given by:

$$\delta = P/(8\rho_w g L^2). \qquad (8\text{-}9)$$

b. For example, let us again consider a 30.5-centimeter-thick (12-inch-thick) ice sheet with a load P equal to 25.67 kN (5772 lbf). The short-term deflection at the load is estimated by assuming the effective elastic modulus to be 690 MPa (10^5 psi), and we get L= 3.67 meters (12 feet), and $\delta = (25{,}670)/(8 \times 9806.6 \times 3.67^2) = 0.024$ meters (6.95 inches), which is almost equal to $0.08h$, the freeboard of the ice sheet. If this load were to be left on that ice sheet for any length of time, the permanent deflection after the elastic deflection takes place would cause the top surface to be below the water surface, creating the possibility of water seeping through cracks and flooding the loaded area. To estimate the long-term deflection, we assume the effective elastic modulus $E = 55$ MPa, and we get $L = 1.95$ meters. Limiting the maximum deflection δ to the freeboard, which is assumed to be 0.08 times the ice thickness, we get an estimate of the long-term load for the ice sheet $P_{\text{long-term}} = 8\rho_w g L^2(0.08h)$. = (8)(1000 kg/m^3)(9.81 m/s^2)(3.7 m)^2(0.08×0.305 m) = 7.3 kN (1640 lbf). These estimates of the loads indicate that the long-term load is about three to four times smaller than the short-term load on an ice sheet.

c. According to available field observations, limiting the maximum deflection of a statically loaded ice cover to the freeboard results in a safe condition. In a field situation, one needs to continuously monitor the remaining freeboard of an ice sheet for long-term storage of a load. A recommended field practice is to drill a hole in the ice sheet near the load and check the freeboard, which is the distance between the water level in the hole and the top surface of the ice sheet. If the water begins to flood the top surface of the ice sheet, it is necessary to move the load immediately to prevent breakthrough attributable to long-term creep deformation of the ice. Another method for predicting the onset of failure is based on the energy method, which requires measurement of the deflection of the ice sheet at the load and keeping a record of the load placed on it. The results of such monitoring effort also give estimates of the safe storage time.

8-8. References

a. Required Publications.

None.

b. Related Publications.

Safe Loads on Ice Sheets 1996
Ice Engineering Exchange Bulletin, Number 13, U.S. Army Cold regions Research and Engineering Laboratory, Hanover, New Hampshire

Ashton 1986
Ashton, G.D., ed. 1986. *River and Lake Ice Engineering*, Water Resources Publications, Littleton, Colorado.